

## Electric Field

“The field around a charge is called electric field”.

Or

“The electric force per unit charge is called electric field, it is also called electric intensity”.

→ It is vector quantity.

$$\vec{E} = \frac{\vec{F}}{q_0} \quad (\text{electric field}).$$

→ Its unit is N/C.

→ Its direction is always from positive to negative charge.

**Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).**

## Electric Field Due To A Point Charge

To find the electric field due to a charged particle (often called a *point charge*), we place a positive test charge at any point near the particle, at distance  $r$ . From Coulomb's law, the force on the test charge due to the particle with charge  $q$  is

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2} \hat{r}.$$

As previously, the direction of  $\vec{F}$  is directly away from the particle if  $q$  is positive (because  $q_0$  is positive) and directly toward it if  $q$  is negative. We can now write the electric field set up by the particle (at the location of the test charge) as

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \quad (\text{charged particle}).$$

Let's think through the directions again. The direction of  $\vec{E}$  matches that of the force on the positive test charge: directly away from the point charge if  $q$  is positive and directly toward it if  $q$  is negative.

So, if given another charged particle, we can immediately determine the directions of the electric field vectors near it by just looking at the sign of the charge  $q$ . We can find the magnitude at any given distance  $r$  by converting above equation to a magnitude form:

$$E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2} \quad (\text{charged particle}).$$

We write  $|q|$  to avoid the danger of getting a negative  $E$  when  $q$  is negative, and then thinking the negative sign has something to do with direction. Above equation gives magnitude  $E$  only. We must think about the direction separately.

Figure gives a number of electric field vectors at points around a positively charged particle, but be careful. Each vector represents the vector quantity at the point where the tail of the arrow is anchored. The vector is not something that stretches from a “here” to a “there” as with a displacement vector.

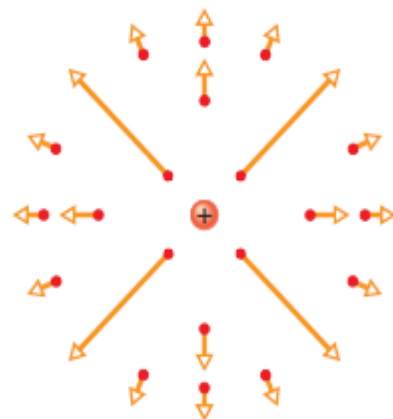
In general, if several electric fields are set up at a given point by several charged particles, we can find the net field by placing a positive test particle at the point and then writing out the force acting on it due to each particle, such as  $F_{01}$  due to particle 1. Forces obey the principle of superposition, so we just add the forces as vectors:

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}.$$

To change over to electric field, we repeatedly use  $E = F/q_0$  for each of the individual forces:

$$\begin{aligned} \vec{E} &= \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0} \\ &= \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n. \end{aligned}$$

This tells us that electric fields also obey the principle of superposition. If you want the net electric field at a given point due to several particles, find the electric field due to each particle (such as  $E_1$  due to particle 1) and then sum the fields as vectors. (As with electrostatic forces, you cannot just willy-nilly add up the magnitudes.)

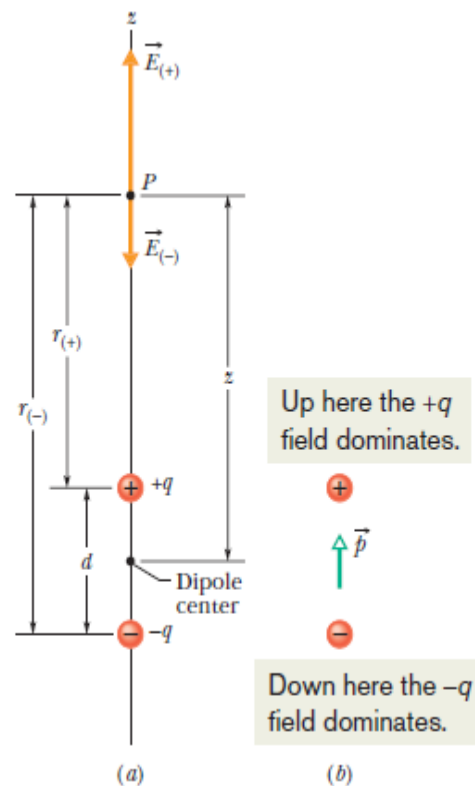


## Electric Field Due To An Electric Dipole

The electric field lines for two particles that have the same charge magnitude  $q$  but opposite signs, a very common and important arrangement known as an **electric dipole**. The particles are separated by distance  $d$  and lie along the *dipole axis*, an axis of symmetry around which you can imagine rotating. Let's label that axis as a  $z$  axis. Here we restrict our interest to the magnitude and direction of the electric field  $E$  at an arbitrary point  $P$  along the dipole axis, at distance  $z$  from the dipole's midpoint.

Figure shows the electric fields set up at  $P$  by each particle. The nearer particle with charge  $+q$  sets up field  $E_{(+)}$  in the positive direction of the  $z$  axis (directly away from the particle). The farther particle with charge  $-q$  sets up a smaller field  $E_{(-)}$  in the negative direction (directly toward the particle). We want the net field at  $P$ . However, because the field vectors are along the same axis, let's simply indicate the vector directions with plus and minus signs, as we commonly do with forces along a single axis. Then we can write the magnitude of the net field at  $P$  as

$$\begin{aligned} E &= E_{(+)} - E_{(-)} \\ &= \frac{1}{4\pi\epsilon_0} \frac{q}{r_{(+)}^2} - \frac{1}{4\pi\epsilon_0} \frac{q}{r_{(-)}^2} \\ &= \frac{q}{4\pi\epsilon_0(z - \frac{1}{2}d)^2} - \frac{q}{4\pi\epsilon_0(z + \frac{1}{2}d)^2}. \end{aligned}$$



After a little algebra, we can rewrite this equation as:

$$E = \frac{q}{4\pi\epsilon_0 z^2} \left( \frac{1}{\left(1 - \frac{d}{2z}\right)^2} - \frac{1}{\left(1 + \frac{d}{2z}\right)^2} \right).$$

After forming a common denominator and multiplying its terms, we come to

$$E = \frac{q}{4\pi\epsilon_0 z^2} \frac{2d/z}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\epsilon_0 z^3} \frac{d}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2}.$$

We are usually interested in the electrical effect of a dipole only at distances that are large compared with the dimensions of the dipole—that is, at distances such that  $z \gg d$ . At such large

distances, we have  $d/2z \ll 1$  in above equation. Thus, in our approximation, we can neglect the  $d/2z$  term in the denominator, which leaves us with

$$E = \frac{1}{2\pi\epsilon_0} \frac{qd}{z^3}.$$

The product  $qd$ , which involves the two intrinsic properties  $q$  and  $d$  of the dipole, is the magnitude  $p$  of a vector quantity known as the **electric dipole moment** of the dipole. (The unit of is the coulomb-meter.) Thus, we can write above equation as

$$E = \frac{1}{2\pi\epsilon_0} \frac{p}{z^3} \quad (\text{electric dipole}).$$

The direction of  $p$  is taken to be from the negative to the positive end of the dipole, as indicated in Fig. We can use the direction of  $p$  to specify the orientation of a dipole.

Above equation shows that, if we measure the electric field of a dipole only at distant points, we can never find  $q$  and  $d$  separately; instead, we can find only their product. The field at distant points would be unchanged if, for example,  $q$  were doubled and  $d$  simultaneously halved. Although above equation holds only for distant points along the dipole axis, it turns out that  $E$  for a dipole varies as  $1/r^3$  for *all* distant points, regardless of whether they lie on the dipole axis; here  $r$  is the distance between the point in question and the dipole center.

Inspection of Fig. and the field lines shows that the direction of for distant points on the dipole axis is always the direction of the dipole moment vector  $p$ . This is true whether point  $P$  in Fig. is on the upper or the lower part of the dipole axis.

Inspection of above equation shows that if you double the distance of a point from a dipole, the electric field at the point drops by a factor of 8. If you double the distance from a single point charge, however, the electric field drops only by a factor of 4. Thus the electric field of a dipole decreases more rapidly with distance than does the electric field of a single charge. The physical reason for this rapid decrease in electric field for a dipole is that from distant points a dipole looks like two particles that almost—but not quite—coincide. Thus, because they have charges of equal magnitude but opposite signs, their electric fields at distant points almost—but not quite—cancel each other.